MATH 54 - HINTS TO HOMEWORK 1

PEYAM TABRIZIAN

Here are a couple of hints to Homework 1! Make sure to attempt the problems before you check out those hints.

1. SECTION 1.1: SYSTEMS OF LINEAR EQUATIONS

1.1.15. All you have to do are row-reductions until it is easier to see whether the equation has a solution or not. In particular, if one of the rows is of the form:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & b \end{bmatrix}$$

then the system has no solution!

1.1.20. Solve the system as if h was a number! It might be useful to divide the second row by -2. Again, use the fact that if one of the rows is of the form:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & b \end{bmatrix}$$

then the system has no solution!
The answer is $\boxed{h = -2}$

1.1.28. The answer is $\boxed{ad - bc \neq 0}$. Solve the system as if a, b, c, d were fixed numbers (say $1, 2, 3, 4$). We'll see later a much easier criterion to solve this problem, namely the determinant of the coefficient matrix, which is $ad - bc$ here has to be nonzero!

Solution: (by popular demand)

Just write the system in matrix form and use row-reductions to solve it:

$$\begin{bmatrix} a & b & f \\ c & d & g \end{bmatrix} \xrightarrow{\div a} \begin{bmatrix} 1 & \frac{b}{a} & \frac{f}{a} \\ c & d & g \end{bmatrix} \xrightarrow{-c} \begin{bmatrix} 1 & \frac{b}{a} & \frac{f}{a} \\ 0 & d - \frac{bc}{a} & g - \frac{fc}{a} \end{bmatrix}$$

In particular, if you want the system to be consistent, there has to be no row of the form:

$$\begin{bmatrix} 0 & 0 & 0 & b \end{bmatrix}$$

So in particular $d - \frac{bc}{a} \neq 0$, so $d \neq \frac{bc}{a}$, so $ad \neq bc$ so $ad - bc \neq 0$.

Another thing you could do is divide the second row by $d - \frac{bc}{a}$, but you'll get the same result because remember that you can't divide by 0.

SECTION 1.2: ROW REDUCTION AND ECHELON FORMS

1.2.15, 1.2.23, 1.2.24, 1.2.25. In each of the problems, the following fact will help you solve the problem:

Fact: A system is consistent if and only if in the row echelon form of the augmented matrix there is no row of the form

$$\begin{bmatrix} 0 & 0 & 0 & \cdots & b \end{bmatrix}$$

Where $b \neq 0$.
For $23, 24, 25$, it'll help to draw a picture of what the matrix in question looks like.

1.2.26. Again, draw a picture of the given matrix. Try out a concrete example to convince you of this! Can you solve for z? If yes, can you solve for y? Finally, can you solve for x?

1.2.30. Underdetermined means 'fewer equations than unknowns'. Find two equations in three unknowns which give you a contradiction, such as $0 = 1$.